Drag force in asymptotically Lifshitz spacetimes

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Abstract

We calculated drag force for asymptotically Lifshitz space times in (d + 2)-dimensions with arbitrary dynamical exponent z. We find that at zero and finite temperature the drag force has a non-zero value. Using the drag force calculations, we investigate the DC conductivity of strange metals.

1 Introduction

A new method for studying different aspects of strongly coupled quantum field theories is the AdS/CFT correspondence [1–4] which has yielded many important insights into the dynamics of strongly coupled field theories. Recently the application of this duality in condensed matter physics called AdS/CMT has been studied [5]. This duality is very useful to study certain strongly coupled systems in CMT by holography techniques and to understand better their properties.

Methods based on AdS/CFT relate gravity in AdS_{d+2} space to the conformal field theory on the $(d+1)$-dimensional boundary. These conformal field theories are invariant under the following scaling transformation

$$(t, \vec{x}) \to (\lambda\, t, \lambda\, \vec{x}), \tag{1.1}$$

However, in many condensed matter systems there are field theories with anisotropic scaling symmetry. This unconventional scaling can be illustrated as

$$(t, \vec{x}) \to (\lambda^z\, t, \lambda\, \vec{x}), \tag{1.2}$$

where z is the dynamical exponent. These field theories exist near a critical phenomena and describe multicritical points in certain magnetic materials and liquid crystals. In the case of $z = 1$, theory benefits from relativistic scale invariance. For $z = 2$, there is a 2+1 dimensional field theory that so-called Lifshitz field theory. This theory has a line of fixed points parameterized by κ and the lagrangian density is given by

$$\mathcal{L} = \int dx^2\, dt \left((\partial_t \phi)^2 - \kappa \left(\nabla^2 \phi \right)^2 \right). \tag{1.3}$$

These fixed points are strongly coupled and they appear in strongly correlated electrons in zero and finite temperature lattice models [8]. Another important theory with unconventional scaling in (1.2) is the theory whose symmetry group is *Schrödinger* group, $Sch(d-1)$ and the geometry is given by a deformation of AdS geometry [6]. Many of the most interesting examples in the condensed matter theory arise in the Lifshitz case.

In this paper we consider massive charge carriers which are described by the flavor branes in the Lifshitz space times. As it was discussed in [9], considering massive charge carriers in this background is the case of interest in modeling of strange metals. Gravity dulas have been investigated in [7].

To calculate the drag force, one should consider a probe brane in this background. Adding flavor branes and finite-density holography have been studied in [11]. These probe branes are related to a quantum critical theory and we consider massive charge carriers interacting with this theory. Notice that these carriers do not backreact on the quantum critical system.

In the massive case, the flavor brane forms a cigar-like shape with its tip at r_0 and charge carriers correspond to strings stretching from the tip of the cigar down to the horizon. At finite temperature, one should consider a black brane in the background at a finite radial position r_h. As a result, charge carriers on the flavor brane correspond to the stretching strings from r_0 to the horizon. To calculate the drag force, one should consider a stretching string from the flavor brane and using prescription in [15–17] find energy loss at zero and finite temperature cases.

The most useful application of drag calculations in Lifshitz background has been done in [9]. They study phenomenology of 'strange metalls' and compute the electrical conductivity. We discuss this application of drag force in the last section. Using the drag force calculations, we investigate the DC conductivity of strange metals and derive some results of [9].

This paper is organized as follows. In the next section, we use the proposed solutions in [20,22] and discuss the energy loss of massive charge carriers. We find that they lose energy even at zero temperature. We compare this result with energy loss of particle computed in the case of non-relativistic gravity dual to field theory with Shrodinger CFT symmetry [12]. Also in this case moving particle loses energy at zero temperature. In section three, we consider the Lifshitz background embedded into string theory [28]. We also find here a non-zero drag force at zero temperature. In the last section we discuss computing DC conductivity from drag calculations.

While this paper was in the final stages of preparation, it came to our attention that the drag force in non-relativistic background whose symmetry is *Schorödinger* group has been done in [29]. This calculation extends the results of [12] and confirms a non-zero drag force at zero temperature.

2 Asymptotic Lifshitz space times

In this section we provide backgrounds which are necessary for our discussions. We consider non-relativistic holography in [20]. In this study, Lifshitz geometry is a solution of gravity coupled to a massive vector field. The (d+2)-dimensional spacetime action is

$$S = \frac{1}{16\pi G_{d+2}} \int d^{d+2}x\sqrt{-g}[R - 2\Lambda - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}e^{\lambda\phi}F_{\mu\nu}F^{\mu\nu}]. \tag{2.1}$$

where Λ is the cosmological constant and massless scalar field and abelian gauge field are matter fields of theory. The only non-vanishing components of the field strength is $F_{rt} =$

$qe^{-\lambda\phi}r^{z-d-1}$ and q is related to the charge of the black hole. Based on this action, one finds the asymptotically Lifshitz solution at zero temperature

$$
\begin{aligned}
ds^2 &= L^2(-r^{2z}dt^2 + \frac{dr^2}{r^2} + r^2\sum_{i=1}^{d} dx_i^2), \\
F_{rt} &= qe^{-\lambda\phi}r^{z-d-1}, \quad e^{\lambda\phi} = r^{\lambda\sqrt{2(z-1)d}}, \\
\lambda^2 &= \frac{2d}{z-1}, \quad q^2 = 2L^2(z-1)(z+d), \\
\Lambda &= -\frac{(z+d-1)(z+d)}{2L^2}.
\end{aligned} \tag{2.2}
$$

In this solution, dilaton is not constant. However, exact solutions can be found. To calculate the drag force, one should consider the flavor probe branes in the background (2.2) and study moving massive charge carriers. From gauge-string duality, we consider a trailing open string in the holographic direction. The action of this open string is given by the Nambu-Goto action

$$
S = -T_0 \int d\tau d\sigma \sqrt{-g}. \tag{2.3}
$$

where T_0 is the tension of the string. The coordinates (σ, τ) parameterize the induced metric g_{ab} on the string world-sheet and g is the determinant of the world-sheet metric g_{ab}

$$
-g = -detg_{ab} = (\dot{X}\cdot X')^2 - (X')^2(\dot{X})^2, \tag{2.4}
$$

where $X^\mu(\sigma,\tau)$ is a map from the string world-sheet into space-time, and we define $\dot{X} = \partial_\tau X$, $X' = \partial_\sigma X$, and $V\cdot W = V^\mu W^\nu G_{\mu\nu}$ where $G_{\mu\nu}$ is the metric. The lagrangian density is given by $\mathcal{L} = -T_0\sqrt{-g}$. The string equation of motion is obtained as

$$
\partial_\rho(\frac{\partial\mathcal{L}}{\partial x'}) + \partial_t(\frac{\partial\mathcal{L}}{\partial\dot{x}}) = 0. \tag{2.5}
$$

One has to calculate the canonical momentum densities π_x^0, π_t^0 to find the total energy and momentum of the moving particle in non-relativistic field theory

$$
E = -\int_{\rho_h}^{\rho_0} d\rho\, \pi_t^0, \quad P = -\int_{\rho_h}^{\rho_0} d\rho\, \pi_x^0. \tag{2.6}
$$

2.1 Drag force at zero temperature

Now, we calculate drag force at zero temperature. We consider a moving heavy point particle on the probe flavor brane in d-dimensional space with the following ansatz

$$
t = \tau, \qquad r = \sigma, \qquad x_1 = x = v\,t + \xi(r), \quad x_i = 0 (i \neq 1), \tag{2.7}
$$

One finds from the equations of motion (2.5) that

$$\xi'^2 = \frac{C^2\left(r^{2z-2}-v^2\right)}{r^{2z+2}\left(r^{2z+2}-C^2\right)}, \tag{2.8}$$

where C is the constant of motion. The drag force that experiences by moving particle is

$$F_{drag} = -T_o v^{2\left(\frac{z+1}{z-1}\right)}. \tag{2.9}$$

The drag force is independent of dimension of space. In (2.8), we considered string from boundary to infinity, then one finds from numerator that there is no bound on the velocity and it can change from zero to infinity. This is because of the fact that dual theory is non-relativistic. This is an interesting result because even though the system is at zero temperature the moving particle losses its energy. We consider mass and momentum of particle as M and P, respectively. Then $P = Mv$ and in the case of constant momentum, the drag force will be found as $F_{drag} = \mu M v$. In this way, one finds friction term as $\mu = \frac{T_0}{M} v^{\left(\frac{z+3}{z-1}\right)}$.

The Author in [12] found a non-zero drag force at zero temperature. They studied non-relativistic three dimensional CFT at zero and finite temperature. They found that unlike the AdS case where one only gets a casual speed limit, in the non-relativistic case one arrives no speed limit and non-zero drag force. We also find same results in asymptotically Lifshitz spacetimes with arbitrary critical exponent. As a result the non-zero drag force would be considered as a common properties of non-relativistic spacetimes.

2.2 Drag force at finite temperature

Unlike the case of *Schrödinger* conformal group, it is difficult to obtain analytic black hole solutions in Lifshitz spacetimes. Actually, the problem of finding analytic exact black hole solutions with asymptotically Lifshitz geometry turned out to be a highly non-trivial problem. However, there are known solutions. For example, black hole solutions with z = 2 in four dimensions were studied in [14] and black holes in asymptotically Lifshitz spacetimes with arbitrary critical exponent were investigated in [18]. Topological black holes and other solutions were proposed in [19,20]. For other recent solutions on Lifshitz black holes see [24].

We consider the black brane solution of (2.1) in asymptotic Lifshitz (2+1)-dimensional spacetime proposed in [20]

$$ds^2 = L^2\left(-r^{2z} f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\vec{x}_i^2\right), \quad f(r) = 1 - \left(\frac{r_+}{r}\right)^{z+d}. \tag{2.10}$$

The matter fields are a massless scalar and an abelian gauge field and the other fields remain the same as those in the zero temperature case. The temperature is

$$T_H = \frac{(z+d) r_+^z}{4\pi}, \tag{2.11}$$

and the black hole entropy is given by

$$S_B = \frac{V_d L^d r_+^d}{4G_{d+2}}, \tag{2.12}$$

which V_d is the volume of the d dimensional spatial coordinates $\vec{x}_i$.

We consider the moving heavy point particle in x direction and following the ansatz in (2.7), one finds that

$$\xi'^2 = \frac{C^2 \left(r^{2z-2} - \frac{v^2}{f(r)}\right)}{f(r) r^{2z+2} \left(L^4 f(r) r^{2z+2} - C^2\right)}, \tag{2.13}$$

where C is constant of motion. From reality condition of above equation, one finds that $C = vL^2 r_c^2$ and r_c is the root of this equation

$$\left(1 - \left(\frac{r_+}{r_c}\right)^{z+d}\right) r_c^{2z-2} - v^2 = 0, \tag{2.14}$$

The drag force is given by

$$F_{drag} = -T_0\, vL^2 r_c^2. \tag{2.15}$$

We consider some special cases and derive the drag force as the following

- $d = 2, z = 1$
  In this case, there are two dimensional spatial coordinates and $z = 1$ means the isotropic theory. The drag force in terms of the temperature of the field theory, $T_H = \frac{3r_+}{4\pi}$, is given by

$$F_{drag} = -T_0 L^2 \left(\frac{4\pi}{3}\right)^{\frac{2}{3}} T_H^{\frac{2}{3}} \frac{v}{(1-v^2)^{\frac{2}{3}}}. \tag{2.16}$$

- $d = 2, z = 2$
  This specific example is known as the Lifshitz model. This case appears in systems of strongly correlated electrons in condensed matter physics. Having a holographic description for these phenomena would be of great importance to investigate some properties of strongly coupled systems in condensed matter. One finds drag force as

$$F_{drag} = -T_0 L^2 v^2 \left(\frac{v^2}{2} - \sqrt{\frac{v^2}{2} + \pi^2\, T_H^2}\right). \tag{2.17}$$

It is clear that in each case the temperature dependency of drag force is different. It is straightforward to discuss drag force in other cases with spatial dimensions more than $d = 2$ and different values for z.

2.3 R^2 corrections to the drag force

Now we study R^2 corrections. These corrections to five-dimensional asymptotically Lifshitz spacetimes have been studied in [22]. The specific example is Gauss- Bonnet model. Black brane solution has been found perturbatively as

$$ds^2 = L^2[-g(u)(1-u)dt^2 + \frac{1}{h(u)(1-u)}du^2 + \frac{r_+^2}{u^A}(dx_1^2 + dx_2^2 + dx_3^2)], \tag{2.18}$$

where

$$\begin{aligned} g(u) &= r_+^{2z} u^{-\frac{4z}{z_0+3}}(1+u)\left(1+\lambda_{\rm GB}(1-u^2)\right)\exp[4\lambda_{\rm GB}\frac{z_0-1}{z_0+3}u^2], \\ h(u) &= \frac{1}{4}(z_0+3)^2 u^2 (1+u)\left(1+\lambda_{\rm GB}(1-u^2)\right). \end{aligned} \tag{2.19}$$

where λ_{GB} is the Gauss-Bonnet coupling constant. and

$$u^2 = (\frac{r_+}{r})^{z_0+3} \quad A = \frac{4}{z_0+3}, \quad z = z_0 + 2\lambda_{GB}(z_0-1). \tag{2.20}$$

The horizon of the black brane locates at $u = 1$ and the boundary locates at $u = 0$. the Hawking temperature is

$$T_H = \frac{(z_0+3)r_+^z}{4\pi}\left(1 + 2\lambda_{GB}\left(\frac{z_0-1}{z_0+3}\right)\right). \tag{2.21}$$

The drag force in the case of $z_0 = 1$ has been calculated in [25].

We consider the moving heavy point particle in x direction and following the ansatz in (2.7), one finds that

$$\xi'^2 = \frac{C^2\left(\frac{g(u)}{h(u)} - \frac{r_+^2 v^2}{u^A h(u)(1-u)}\right)}{\frac{r_+^2(1-u)g(u)}{u^A}\left(\frac{L^4 r_+^2(1-u)g(u)}{u^A} - C^2\right)}, \tag{2.22}$$

From reality condition of above equation, We should find roots of this equation

$$g(u_c) - \frac{r_+^2 v^2}{u_c^A(1-u_c)} = 0, \tag{2.23}$$

Regarding this equation, drag force can be found as

$$F_{drag} = -T_0 r_+^2 v\left(\frac{1+u_c}{u_c^A}\right). \tag{2.24}$$

Notice that dependency of drag force to the temperature of the field theory is complicated.

One can study charge effects on the drag force, too. The charged Lifshitz black hole solutions in general (d + 2)- dimensions have been investigated in [23]. Also the Gauss-Bonnet corrections to such black holes in five dimensions have been calculated perturbatively. Using these solutions, one can study drag force in these backgrounds.

3 Drag force in String Duals of Non-relativistic Lifshitz-like Theories

The aim of this section is to study some features of (1+2)-dimensional non-relativistic field theory using supergravity solution in type IIB string theory. Since we are dealing with string theory, it is natural to consider a semi-classical string in this background. However, it is difficult to embed the Lifshitz background into string theory. Some no-go theorems for string theory duals of non-relativistic Lifshitz like theories have been proposed in [26]. They propose that classical solutions in type IIA and eleven-dimensional supergravities are not possible. (These solutions are expected to be dual to (2+1)- dimensional Lifshitz-like theories.) Based on holographic constructions of fractional quantum Hall effect (FQHE) via string theory, authors of [27] proposed D3-D7 solutions. Using this construction, the embedding of anisotropic background into type IIB string theory was studied in [28]. However, the scaling behavior in this solution is different and the anisotropy of the scale transformation is only through one of the three spatial directions. As a result, it corresponds to a classical Lifshitz point. Also since it has a non-constant dilaton, the anisotropic scale invariance only holds at the leading order of interactions. In the context of AdS/CFT correspondence an open string can be associated to a Wilson loop in the dual field theory and one can consider particle at the end of this semi-classical open string at the boundary. Regarding this study we consider a moving point particle in a strongly correlated system and calculate the drag force.

3.1 Drag force at zero temperature

We study drag force at zero temperature. Spacetime metric in the Einstein frame is given by [28]

$$ds_E^2 = \tilde{R}^2 \left[r^2(-dt^2 + dx^2 + dy^2) + r^{\frac{4}{3}} dw^2 + \frac{dr^2}{r^2} \right] + R^2 ds_{X_5}^2, \tag{3.1}$$

This metric is invariant under the scaling

$$(t, x, y, w, r) \to \left(\lambda t, \lambda x, \lambda y, \lambda^{\frac{2}{3}} w, \frac{r}{\lambda} \right), \tag{3.2}$$

and therefore is expected to be holographically dual to Lifshitz-like fixed points with space-like anisotropic scale invariance. One can redefine the radius coordinate $\rho \equiv r^{\frac{2}{3}}$ and rescale (t, x, y, w) accordingly. Then metric in string frame will be as the following

$$ds_E^2 = \tilde{R}^2 \left[\rho^3(-dt^2 + dx^2 + dy^2) + \rho^2 dw^2 + \frac{d\rho^2}{\rho^2} \right] + R^2 ds_{X_5}^2. \tag{3.3}$$

This can be regarded as gravity duals of Lifshitz-like fixed points with $z = 3/2$.

Now we study a moving heavy particle in "x" and "w" directions. We expect different behaviors in two directions. Because "w" direction is an anisotropic direction but "x" is not.

Moving in x direction, a time dependent solution:

In this case, we study moving particle in x direction and consider a time dependent solution as the following ansatz

$$X^\mu = (t = \tau, \ \ x = vt + \xi(\rho), \ y = 0, \ w = 0, \ \rho). \tag{3.4}$$

one finds the Nambu-Goto action

$$S = -\frac{1}{2\pi\alpha'} \int dt d\rho \tilde{R}^2 \sqrt{\rho(1 - v^2) + \xi'^2 \rho^6}. \tag{3.5}$$

It would be straightforward to calculate ξ' from the above equation

$$\xi'^2 = \frac{A^2 \rho(1 - v^2)}{\rho^6 \left(\tilde{R}^4 \rho^6 - A^2\right)}, \tag{3.6}$$

where B is the constant of motion. By studying the reality condition for lagrangian density $\mathcal{L} = -T_0\sqrt{-g}$ and therefore for ξ'^2, one finds that the constant of motion can be chosen arbitrary. In the special case of moving with speed of light, one finds $F_{drag} = -T_0\, \tilde{R}^2\, \rho^3$. One observes that there is a bound on the velocity which it was expected because x direction is not anisotropic direction.

Moving in w direction, a time dependent solution

In this case we ask about the moving non-relativistic particle in anisotropic direction. The ansatz is $w = vt + \xi(\rho)$ and from the Nambu-Goto action (2.3), one finds that

$$\xi'^2 = \frac{B^2(\rho - v^2)}{\rho^5(\rho^5 - B^2)}, \tag{3.7}$$

ξ'^2 must be a real parameter and with this condition, B can be found. Also it is clear from the numerator of (3.7) that there is no bound on the velocity of particle and it can be changed from zero to infinity. This result is reasonable, because "w" direction is anisotropic and the dual theory is non-relativistic. As a result, one finds a non-zero drag force on a moving particle in w direction and at zero temperature as

$$F_{drag} = \frac{dp}{dt} = -T_0\, B = -T_0 v^5. \tag{3.8}$$

One can consider the momentum and mass of particle as P and M, respectively and use the non-relativistic relation $P = Mv$. We rewrite drag force in terms of P and derive momentum of particle as

$$P(t) = \left(\frac{4T_0}{M^5}\right)^{\frac{1}{4}} \frac{1}{t^{\frac{1}{4}}}. \tag{3.9}$$

It would be interesting to calculate friction coefficient of moving particle. Using the relation $\dot{P} = -\mu M v$, one finds that friction coefficient is velocity dependent, $\mu = \frac{T_0}{M} v^4$.

In this case although the system is at zero temperature, the moving particle losses its energy. This phenomena is in common with [12] which they consider non-relativistic three dimensional CFT at zero temperature.

3.2 Drag force at finite temperature

It was shown that an AdS space with a black brane is dual to conformal field theory at finite temperature [4]. We use the extension of AdS/CFT correspondence in the case of an anisotropic spacetime. This gravity dual is known as the string theory duals Lifshitz-like fixed points [28]. The metric in the Einstein frame is

$$ds_E^2 = \tilde{R}^2 \left[r^2(-f(r)dt^2 + dx^2 + dy^2) + r^{\frac{4}{3}} dw^2 + \frac{dr^2}{r^2 f(r)} \right] + R^2 ds_{X_5}^2, \tag{3.10}$$

where

$$f(r) = 1 - \frac{\mu}{r^{\frac{11}{3}}}. \tag{3.11}$$

The constant μ represents the mass parameter of the black brane. and the Hawking temperature is

$$T_H = \frac{11}{12\pi} \mu^{\frac{3}{11}}. \tag{3.12}$$

We study a moving object in the hot gauge theory and in the bulk space a moving open string should be considered, too.

Based on the Nambu-Goto action of string in (2.3) and equation of motion in (2.5) the simplest solution for the equation of motion is $x = constant$. In this case the string is stretched from the probe D-brane at $\rho = \rho_m$ to the horizon at $\rho = \rho_h$, straightforwardly. In the other word we have a static particle without any motion. The energy of particle in this case is associated with the rest mass of the particle which is obtained by

$$M_{rest} = \frac{T_0 \tilde{R}^2}{2} \left(\rho_m^2 - \rho_h^2\right). \tag{3.13}$$

One can compare this result with the rest mass of a static quark in $\mathcal{N} = 4$ SYM theory [15]. In our study, one may use the relation $E_{gap} = MC^2$ and interpret (3.13) as the energy scale of bulk excitations at the position of flavor brane.

Moving in x direction, a time dependent solution:

We consider the moving particle in x direction and consider a time dependent ansatz $x = vt + \xi(r)$, with this choice, one finds that

$$\xi'^2 = \frac{1 - \frac{v^2}{f(r)}}{r^4 f(r)\left(\tilde{R}^4 r^4 f(r) - C^2\right)}, \tag{3.14}$$

from this equation, we can find the critical radius where numerator and denominator change their sign

$$r_c = \left(\frac{\mu}{1 - v^2}\right)^{\frac{3}{11}}, \tag{3.15}$$

and finally the drag force on the particle is given by

$$\frac{dp}{dt} = -(\frac{12\,\pi}{11})^2\,\tilde{R}^2\,T_H^2\,\frac{v}{(1 - v^2)^{\frac{6}{11}}}. \tag{3.16}$$

And it is clear that the moving particle loses its energy at finite temperature case, too.

Moving in w direction, a time dependent solution:

It would be interesting to study a moving object in "w" direction. This is an anisotropic direction which spacetime violates lorentz symmetry. One can consider the following ansatz

$$X^{\mu} = (t = \tau,\;\; x = 0,\;\; y = 0,\;\; w = vt + \xi(r),\;\; r). \tag{3.17}$$

The constant of motion can be considered as D and from equation of motion, one finds that

$$\xi'^2 = \frac{D^2(1 - \frac{v^2\,r^{\frac{-2}{3}}}{f(r)})}{r^{\frac{10}{3}} f(r)\left(\tilde{R}^4 r^{\frac{10}{3}} f(r) - D^2\right)}, \tag{3.18}$$

Both numerator and denominator must change sign at the same root, and from numerator one should solve $r^{\frac{11}{3}} - v^2 r^3 - \mu = 0$ to find drag force. We name the root as r_c and one finds drag force as

$$F_{drag} = -T_0\,v\,r_c^{\frac{4}{3}}. \tag{3.19}$$

Based on this relation, one can discuss on the drag force, too.

4 Discussion

In this paper we have calculated drag force for asymptotically Lifshitz space times in (d + 2)-dimensions with arbitrary dynamical exponent z from gauge-string duality. We have used the proposed solutions in [20, 22]. The finite temperature behavior of Wilson loops as an application to strongly coupled gauge theories in 3+1 dimensions has been studied in [13, 14]. By analyzing action in (1.3), one concludes that the boundary theory can be viewed as a gauge theory in 2+1 dimensions with a dimensionless coupling constant and as a result the theory perhaps has some features in common with conventional gauge theory in 3 + 1 dimensions [14]. Having Wilson loops on the gravity side, one can study drag force in the gauge theory side. From gauge-string duality, one should consider a hanging string from the boundary to the horizon. The end point of the string represents the particle that is charged under the gauge field. We have considered a moving heavy point particle and calculated drag force at zero and finite temperature non-relativistic field theories.

We have found the energy loss of moving heavy point particle. For the zero temperature background, we found that particle loses energy even though at zero temperature. Also we considered the string theory dual to Lifshitz-like fixed points with anisotropic scale invariance which proposed in [28] and studied drag force. We found a non-zero drag force in the case of zero temperature, too. In this case, there are anisotropic and isotropic directions. We have found a non-zero drag force when particle is moving in these directions in (3.8). Then, we compared our results with drag force in the case of field theory whose symmetry group is *Schrödinger* group in [12]. In this reference, the energy loss of particle computed in the case of non-relativistic gravity dual to field theory with *Schrödinger* CFT symmetry. Also they found that a moving particle loses energy at zero temperature. Based on these studies, we conclude that this could be a common property of non-relativistic field theories. Holographic description of strongly correlated systems in condensed matter physics implies a non-zero drag force on a moving heavy carrier at zero and finite temperature.

The most useful application of drag calculations in Lifshitz background has been done in [9]. They study phenomenology of 'strange metalls' and compute the electrical conductivity. It would be interesting to relate our results to calculation of DC conductivity.

In order to calculate DC conductivity, an electric field should be turn on on the D-brane probe and the resultant current J^x can be computed in the boundary [10]. The conductivity $\sigma(E,T)$ is found from Ohm's low

$$\sigma(E,T) = \sqrt{\sigma_o^2 + \sigma^2} \tag{4.1}$$

where σ_0 is a constant term and arises from thermally produced pairs of charge carriers.

By increasing the mass of carriers, σ_0 can be made arbitrary small and the leading term in conductivity will be σ. Based on the results of [10], we discuss calculating of leading term in conductivity, σ, by studying the properties of a moving single string. The Authors in [9] found that

$$\sigma^2 = \left(\frac{2\pi\alpha'}{L^2}\right)^2 r_*^{-4} \, (J^t)^2 \tag{4.2}$$

where r_* is the root of the following equation

$$r^{2z+2} \, f(r) - (2\pi\alpha')^2 \, E^2 = 0. \tag{4.3}$$

An important result of [9] is based on the relation (4.2). This equation exhibits the power-law for the DC resistivity, $\rho \sim \frac{T^{\frac{2}{z}}}{J^t}$. As it was discussed in [9], this behavior is generic in a regime of dilute charge carriers which are coupled to a Lifshitz matter. Now we derive this result from our drag force calculations.

We consider the quasi-particle description and write the equation of motion for them at the equilibrium where the external force is $f = E$. At large mass limit, only charge carriers contribute to current and one may express J^x in terms of velocity of the quasi-particles, $J^x = J^t \, v$. Regarding Ohm's law($J^x = \sigma \, E$) one finds the leading term in conductivity as

$$\sigma = \frac{v \, J^t}{E}. \tag{4.4}$$

Based on the drag force calculations at finite temperature and from numerator of (2.13), one finds velocity of the quasi-particle as

$$v^2 = r_*^{2z-2} \, f(r_*) \tag{4.5}$$

The drag force is related to the constant of motion C which can be found from denominator of (2.13). Also at equilibrium, $C^2 = (2\pi\alpha')^2 \, E^2$ then

$$E^2 = \frac{L^4 \, r_*^{2z+2} \, f(r_*)}{(2\pi\alpha')^2} \tag{4.6}$$

From (4.5) and (4.6), one derives conductivity as

$$\sigma = \frac{2\pi\alpha'}{L^2} r_*^{-2} J^t \tag{4.7}$$

which is the same as (4.2). When external field is very weak, one concludes that $r_* \sim r_+$ and as a result $\sigma \sim \frac{J^t}{T^{\frac{2}{z}}}$ which confirms the result of [9].

It would be interesting to study R^2 corrections to the DC conductivity for asymptotically Lifshitz backgrounds. One should consider calculation of the drag force from (2.22). It is

straightforward to find $\sigma_{R^2} = \frac{2\pi\alpha'}{L^2} r_*^{-2} J^t$ and for small electric field one concludes that $r_* \sim r_+$ where r_+ is radial horizon and it is related to the temperature of the matter (2.21). As a result R^2 corrections to conductivity is given by

$$\sigma_{R^2} \sim \left(\frac{4\pi}{z_0 + 3 + 2\lambda_{GB}(z_0 - 1)} \right)^{-2/z} T_H^{-2/z} J^t. \tag{4.8}$$

From this equation, One can study the effect of Gauss-Bonet coupling constant λ_{GB} on the DC conductivity and resistivity. In the case of $z_0 = 1$ one finds the result of [9].

Calculating DC conductivity at zero temperature is straightforward. One should find the velocity of massive charge from numerator of (2.8) and based on the relation between velocity and conductivity, one finds

$$\sigma_{T=0} = \frac{2\pi\alpha'}{L^2} r_c^{-2} J^t. \tag{4.9}$$

where r_c is not the same as r_* in (4.7).
It is interesting to investigate calculation of conductivity from drag force in type IIB string theory which it was studied in [28]. We consider the finite temperature matter and turn on an small electric field in "x" and "w" directions. In these directions one finds that $\sigma_x \sim \frac{1}{T^2}$ and $\sigma_w \sim \frac{1}{T^{4/3}}$. As we expected, conductivity is different in these directions.

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